

CHAPTER 9

Time Value Analysis

- **Future and present values**
 - **Lump sums**
 - **Annuities**
 - **Uneven cash flow streams**
- **Solving for I and N**
- **Investment returns**
- **Amortization**

Time Value of Money

- Time value analysis is necessary because money has **time value**.
 - A dollar in hand today is *worth more* than a dollar to be received in the future.
Why?
 - Because of time value, the values of future dollars must be *adjusted* before they can be compared to current dollars.
- **Time value analysis** constitutes the techniques that are used to account for the time value of money.

Time Lines



Tick marks designate *ends* of periods. Time 0 is the starting point (the beginning of Period 1); Time 1 is the end of Period 1 (the beginning of Period 2); and so on.

What is the FV after 3 years of a \$100 lump sum invested at 10%?



- Finding future values (moving to the *right* along the time line) is called **compounding**.
- For now, assume interest is paid *annually*.

After 1 year:

$$\begin{aligned}FV_1 &= PV + INT_1 = PV + (PV \times I) \\ &= PV \times (1 + I) \\ &= \$100 \times 1.10 = \mathbf{\$110.00}.\end{aligned}$$

After 2 years:

$$\begin{aligned}FV_2 &= FV_1 + INT_2 \\ &= FV_1 + (FV_1 \times I) = FV_1 \times (1 + I) \\ &= PV \times (1 + I) \times (1 + I) = PV \times (1 + I)^2 \\ &= \$100 \times (1.10)^2 = \mathbf{\$121.00}.\end{aligned}$$

After 3 years:

$$\begin{aligned}FV_3 &= FV_2 + I_3 \\ &= PV \times (1 + I)^3 \\ &= 100 \times (1.10)^3 \\ &= \mathbf{\$133.10}.\end{aligned}$$

In general,

$$FV_N = PV \times (1 + I)^N .$$

Three Primary Methods to Find FVs

- Solve the FV equation using a *regular (nonfinancial) calculator*.
- Use a *financial calculator*; that is, one with financial functions.
- Use a *computer* with a *spreadsheet program* such as Excel, Lotus 1-2-3, or Quattro Pro.

Nonfinancial Calculator Solution



$$\mathbf{\$100 \times 1.10 \times 1.10 \times 1.10 = \$133.10.}$$

Financial Calculator Solution

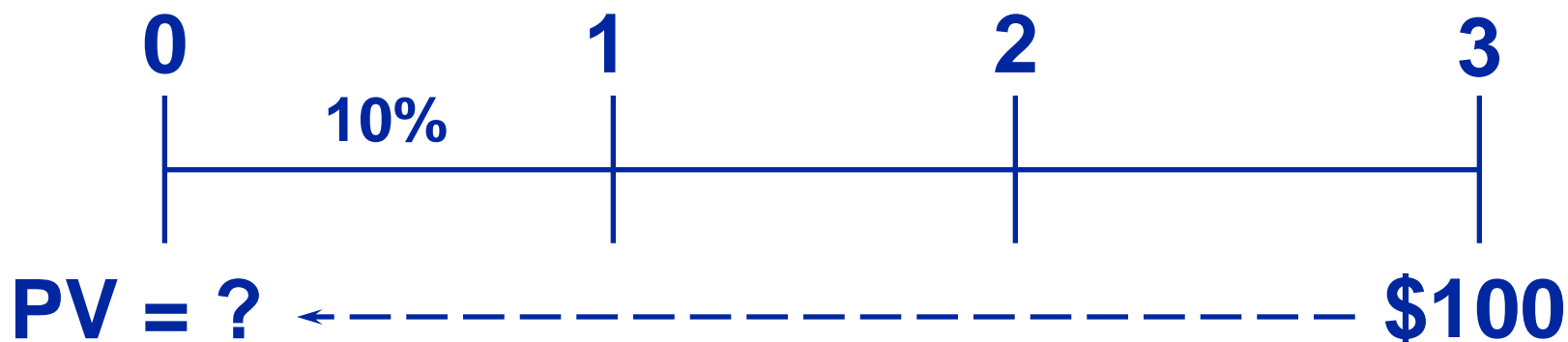
INPUTS	3	10	-100	0	
	N	I/YR	PV	PMT	FV
OUTPUT					133.10

- Notes: (1) Set your calculator on P/YR = 1, END.
- (2) For lump sums, the PMT key is *not* used. Either clear the calculator before you start or enter PMT = 0.

Spreadsheet Solution

	A	B	C	D
1				
2		3 Nper	Number of periods	
3	\$ 100.00	Pv	Present value	
4	10.0%	Rate	Interest rate	
5				
6	\$ 133.10	=100*(1.10)^3 (entered into Cell A6)		
7				
8	\$ 133.10	=A3*(1+A4)^A2 (entered into Cell A8)		
9				
10	\$ 133.10	=FV(A4,A2,,-A3) (entered into Cell A10)		

What is the PV of \$100 due
in 3 years if $i = 10\%$?



Finding present values (moving to the *left* along the time line) is called **discounting**.

Solve $FV_N = PV \times (1 + I)^N$ for PV

$$PV = FV_N / (1 + I)^N.$$

$$\begin{aligned} PV &= \$100 / (1.10)^3 \\ &= \$100(0.7513) = \mathbf{\$75.13}. \end{aligned}$$

Time Line Solution



$$\$100 \div 1.10 \div 1.10 \div 1.10 = \$75.13.$$

Note that the calculated present value (**\$75.13**), when invested at **10** percent for **three** years, will produce the starting future value (**\$100**).

Financial Calculator Solution

INPUTS

3

10

0

100

N

I/YR

PV

PMT

FV

OUTPUT**-75.13**

Either PV or FV must be *negative* on most calculators. Here, $PV = -75.13$. Put in \$75.13 today, take out \$100 after three years.

Spreadsheet Solution

	A	B	C	D
1				
2		3 Nper	Number of periods	
3	\$ 100.00	Fv	Future value	
4	10.0%	Rate	Interest rate	
5				
6	\$ 75.13	=A3/(1+A4)^A2 (entered into Cell A6)		
7				
8	\$ 75.13	=PV(A4,A2,,-A3) (entered into Cell A8)		
9				
10				

Opportunity Cost Rate

- On the last illustration we needed to apply a **discount rate**. Where did it come from?
 - The discount rate is the **opportunity cost rate**.
 - It is the rate that could be earned on alternative investments of *similar risk*.
 - It does *not* depend on the source of the investment funds.
- We will apply this concept over and over in this course.

Opportunity Cost Rate (Cont.)

- The opportunity cost rate is found (at least in theory) as follows:
 - Assess the riskiness of the cash flow(s) to be discounted.
 - Identify *security investments* that have the same risk. **Why securities?**
 - Estimate the return expected on these similar-risk investments.
- When applied, the resulting PV provides a return *equal* to the opportunity cost rate.
- In most time value situations, benchmark opportunity cost rates are known.

Solving for I

Assume that a bank offers an account that will pay **\$200** after **five** years on each **\$75** invested. What is the implied interest rate?

INPUTS

5

N

-75

PV

0

PMT

200

FV

OUTPUT

21.7

Spreadsheet Solution

	A	B	C	D
1				
2	5	Nper	Number of periods	
3	\$ (75.00)	Pv	Present value	
4	\$ 200.00	Fv	Future value	
5				
6				
7				
8	21.7%	=RATE(A2,,A3,A4) (entered into Cell A8)		
9				
10				

Solving for N

Assume an investment earns **20** percent per year. How long will it take for the investment to double?

INPUTS		20	-1	0	2
	N	I/YR	PV	PMT	FV
OUTPUT	3.8				

? What is the **Rule of 72**?

Spreadsheet Solution

	A	B	C	D
1				
2	20.00%	Rate	Interest rate	
3	\$ (1.00)	Pv	Present value	
4	\$ 2.00	Fv	Future value	
5				
6				
7				
8	3.80	=NPER(A2,,A3,A4) (entered into Cell A8)		
9				
10				

Types of Annuities

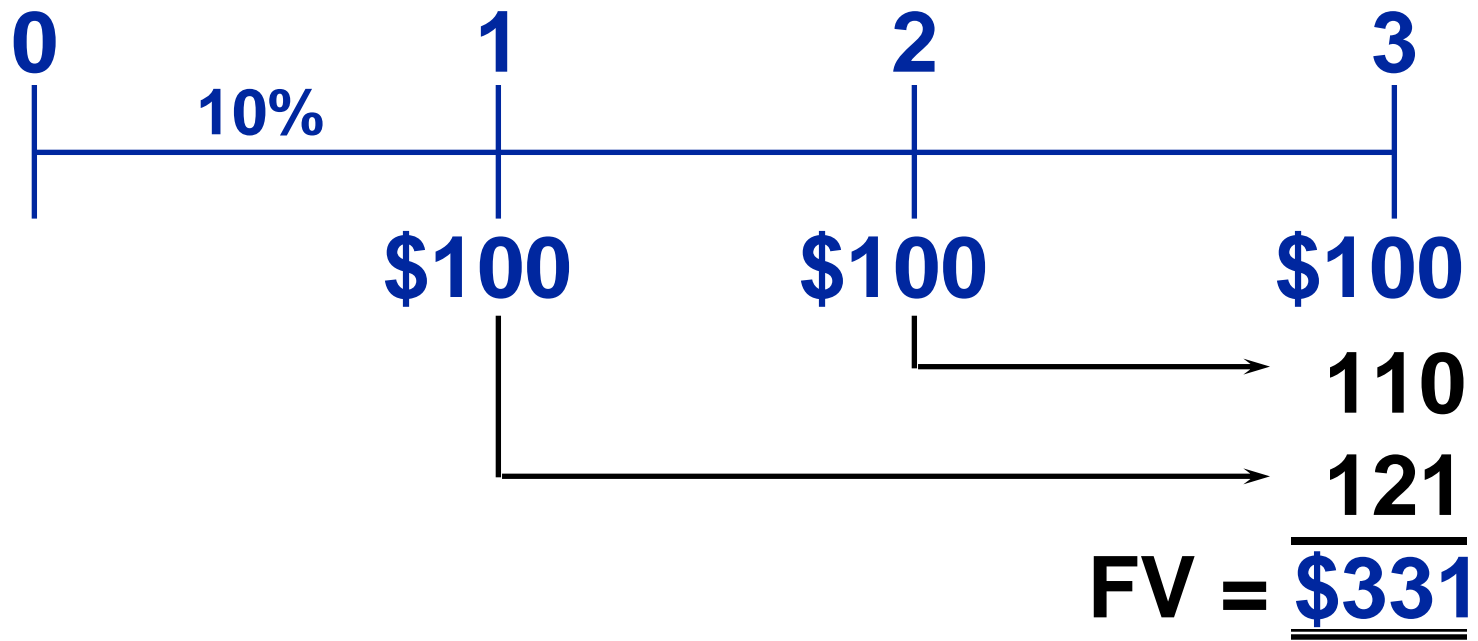
Three-Year **Ordinary Annuity**



Three-Year **Annuity Due**



What is the FV of a three-year ordinary annuity of \$100 invested at 10%?



Financial Calculator Solution

INPUTS

3

10

0

-100

N

I/YR

PV

PMT

FV

OUTPUT

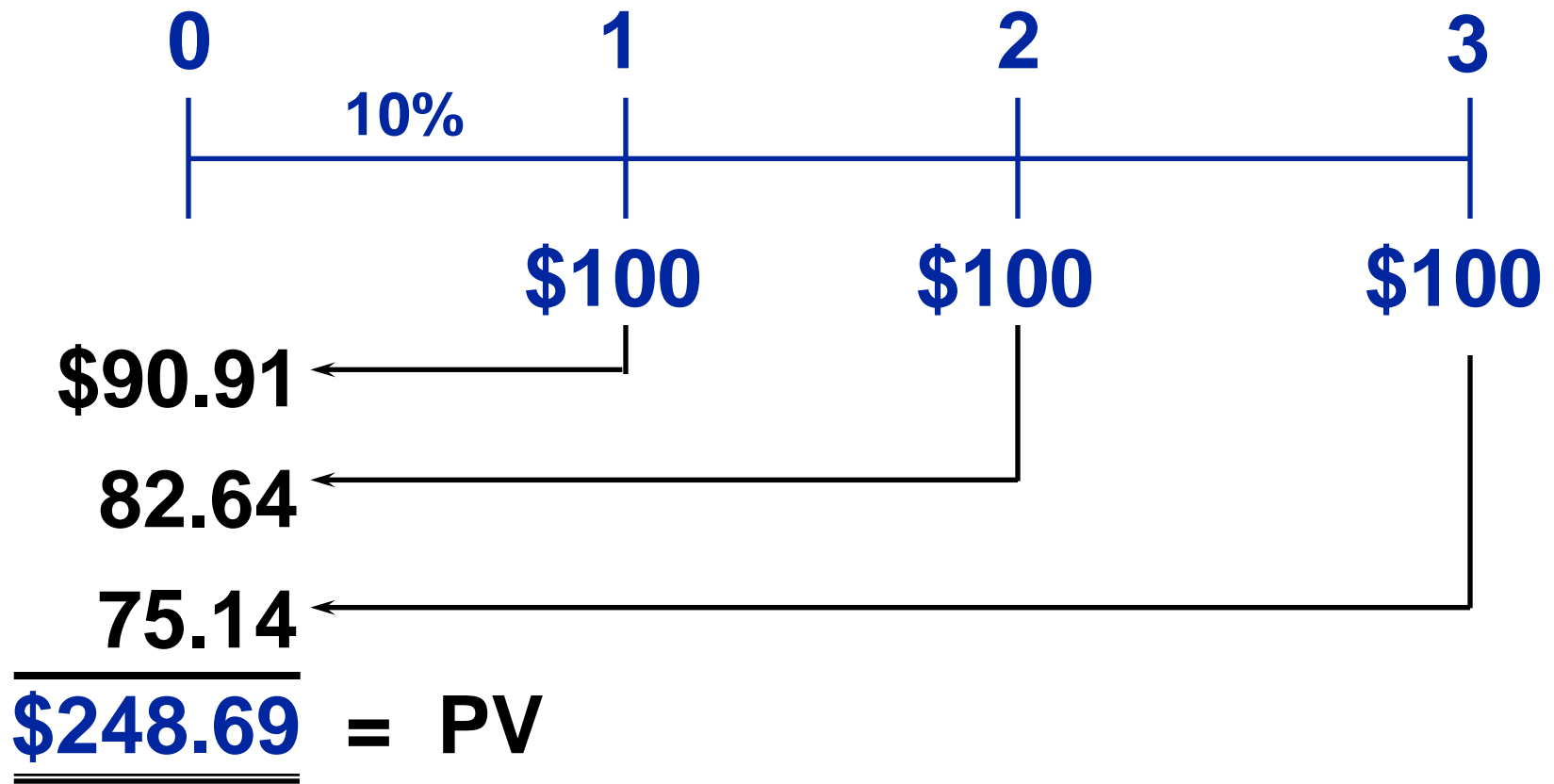
331.00

Here there are payments rather than a lump sum present value, so enter **0** for PV.

Spreadsheet Solution

	A	B	C	D
1				
2		3 Nper	Number of periods	
3	\$ (100.00)	Pmt	Payment	
4	10.0%	Rate	Interest rate	
5				
6				
7				
8	\$ 331.00	=FV(A4,A2,A3) (entered into Cell A8)		
9				
10				

What is the PV of the annuity?



Financial Calculator Solution

INPUTS

3

N

10

I/YR

100

PMT

0

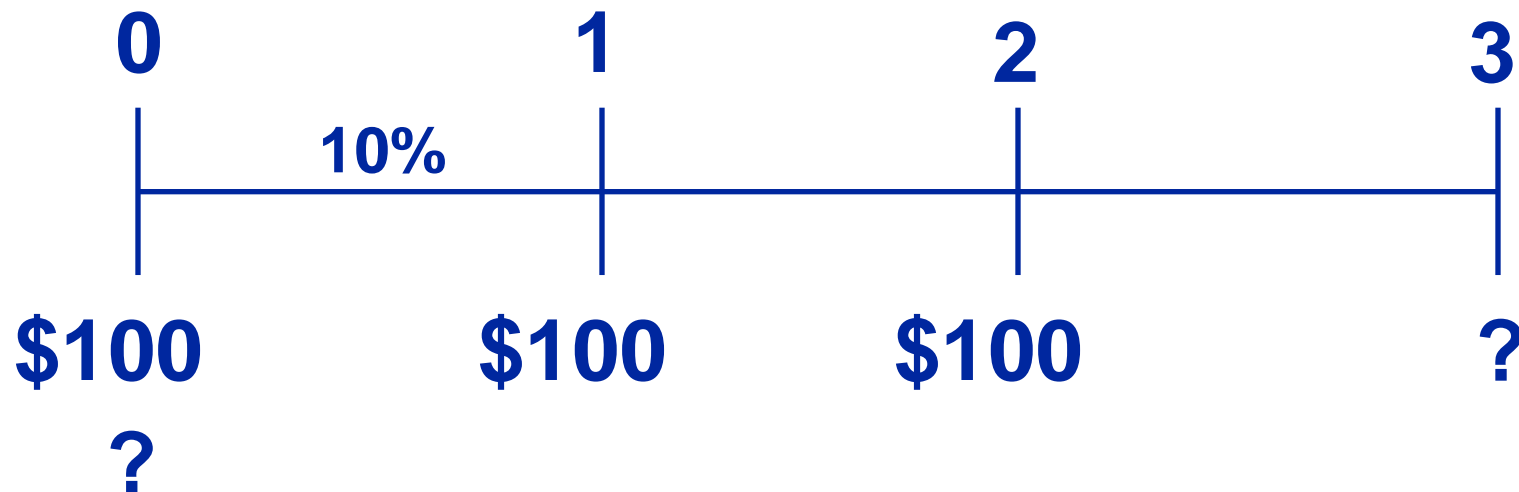
FV

OUTPUT**-248.69**

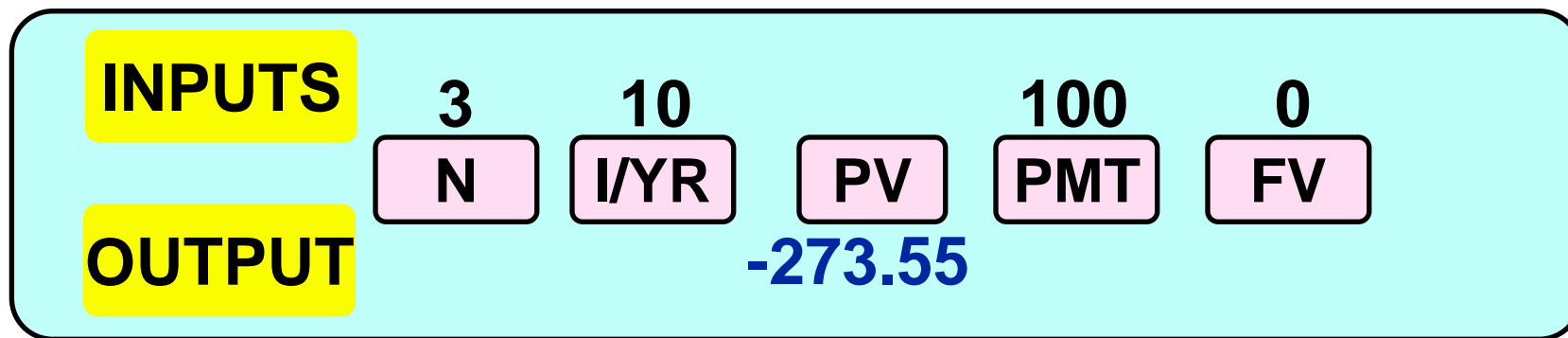
Spreadsheet Solution

	A	B	C	D
1				
2		3 Nper	Number of periods	
3	\$ (100.00)	Pmt	Payment	
4	10.0%	Rate	Interest rate	
5				
6				
7				
8	\$ 248.69	=PV(A4,A2,A3) (entered into Cell A8)		
9				
10				

What is the FV and PV if the annuity were an annuity due?



Switch from *End* to *Begin* mode on a financial calculator. Repeat the annuity calculations. First find $PVA_3 = \$273.55$.



Then enter $PV = 0$ and press **FV** to find $FV = \$364.10$.

Spreadsheet Solution

	A	B	C	D
1				
2	3	Nper	Number of periods	
3	\$ (100.00)	Pmt	Payment	
4	10.0%	Rate	Interest rate	
5				
6	\$ 273.55	=PV(A4,A2,A3,,1) (entered into Cell A6)		
7				
8	\$ 273.55	=PV(A4,A2,A3)*(1+A4) (entered into Cell A8)		
9				
10				

Spreadsheet Solution

	A	B	C	D
1				
2		3 Nper	Number of periods	
3	\$ (100.00)	Pmt	Payment	
4	10.0%	Rate	Interest rate	
5				
6	\$ 364.10	=FV(A4,A2,A3,,1) (entered into Cell A6)		
7				
8	\$ 364.10	=FV(A4,A2,A3)*(1+A4) (entered into Cell A8)		
9				
10				

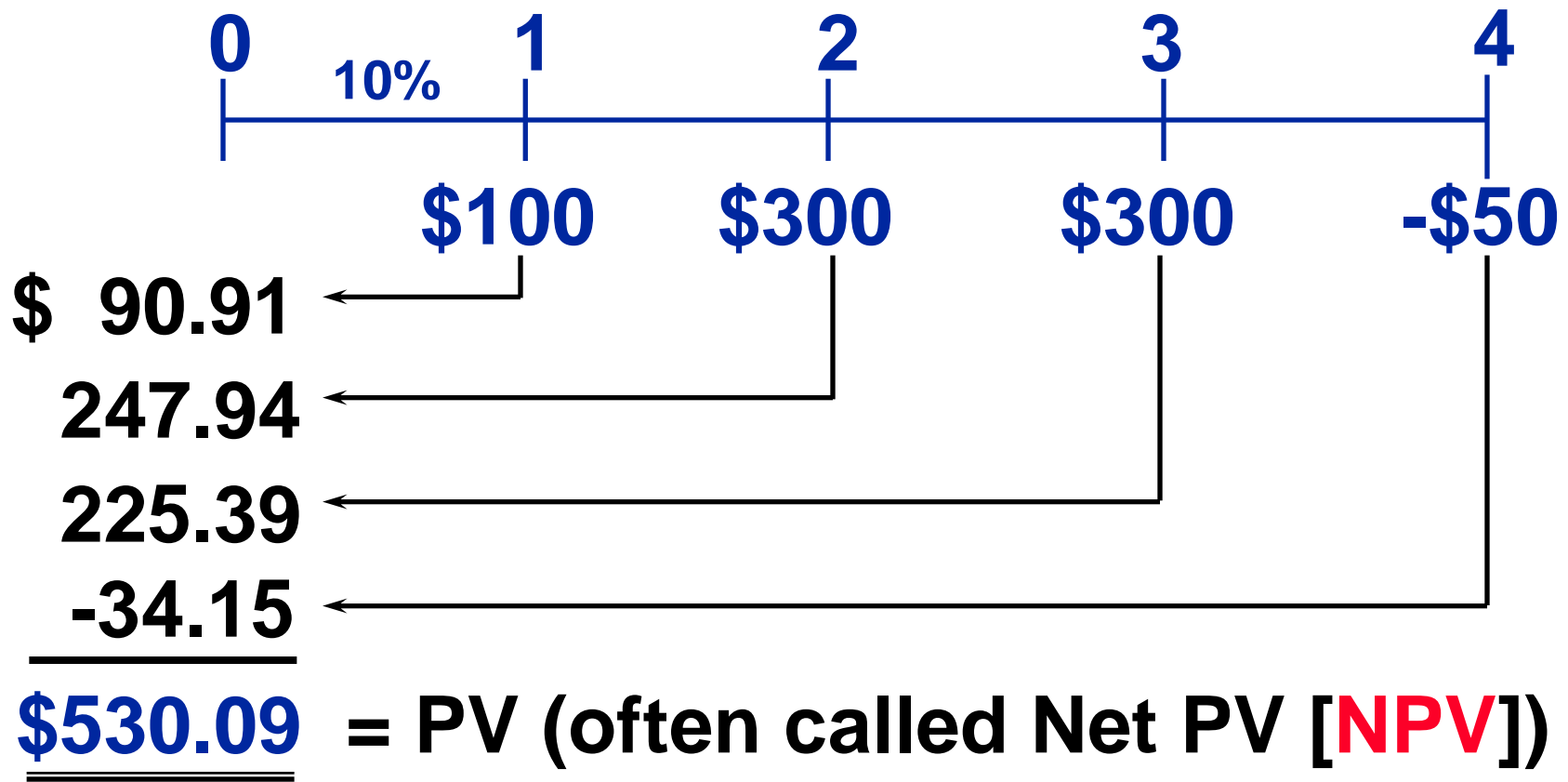
Perpetuities

- A **perpetuity** is an annuity that lasts forever.
- What is the present value of a perpetuity?

$$PV (\text{Perpetuity}) = \frac{PMT}{i} .$$

- ? What is the future value of a perpetuity?

Uneven Cash Flow Streams



Spreadsheet Solution

	A	B	C	D
1				
2	10.0%	Rate	Interest rate	
3				
4	\$ 100	Value 1	Year 1 CF	
5	300	Value 1	Year 2 CF	
6	300	Value 1	Year 3 CF	
7	(50)	Value 1	Year 4 CF	
8				
9				
10	\$ 530.09	=NPV(A2,A4:A7) (entered into Cell A10)		

Discussion Items

Assume the cash flows on the previous slide are the cash flows from an investment. How much would you be willing to pay for these flows?

What would be the financial benefit to you if you could buy it for less, say, **\$500?**

Investment Returns

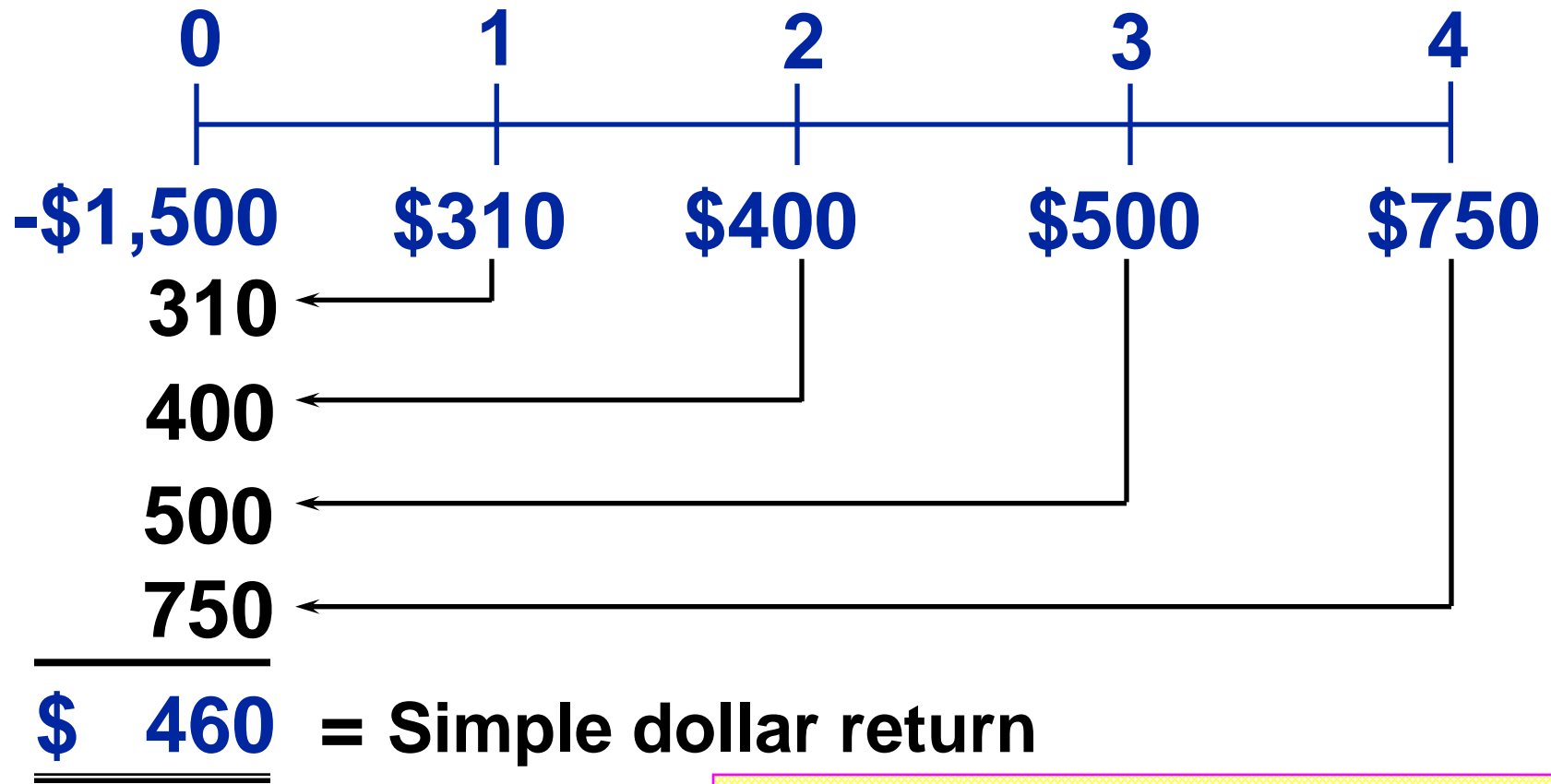
- The financial performance of an investment is measured by its **return**.
 - Time value analysis is used to calculate investment returns.
 - Returns can be measured either in **dollar terms** or in **rate of return** terms.
- Assume that a hospital is evaluating a new MRI. The project's expected cash flows are given on the next slide.

MRI Investment Expected Cash Flows (in thousands of dollars)



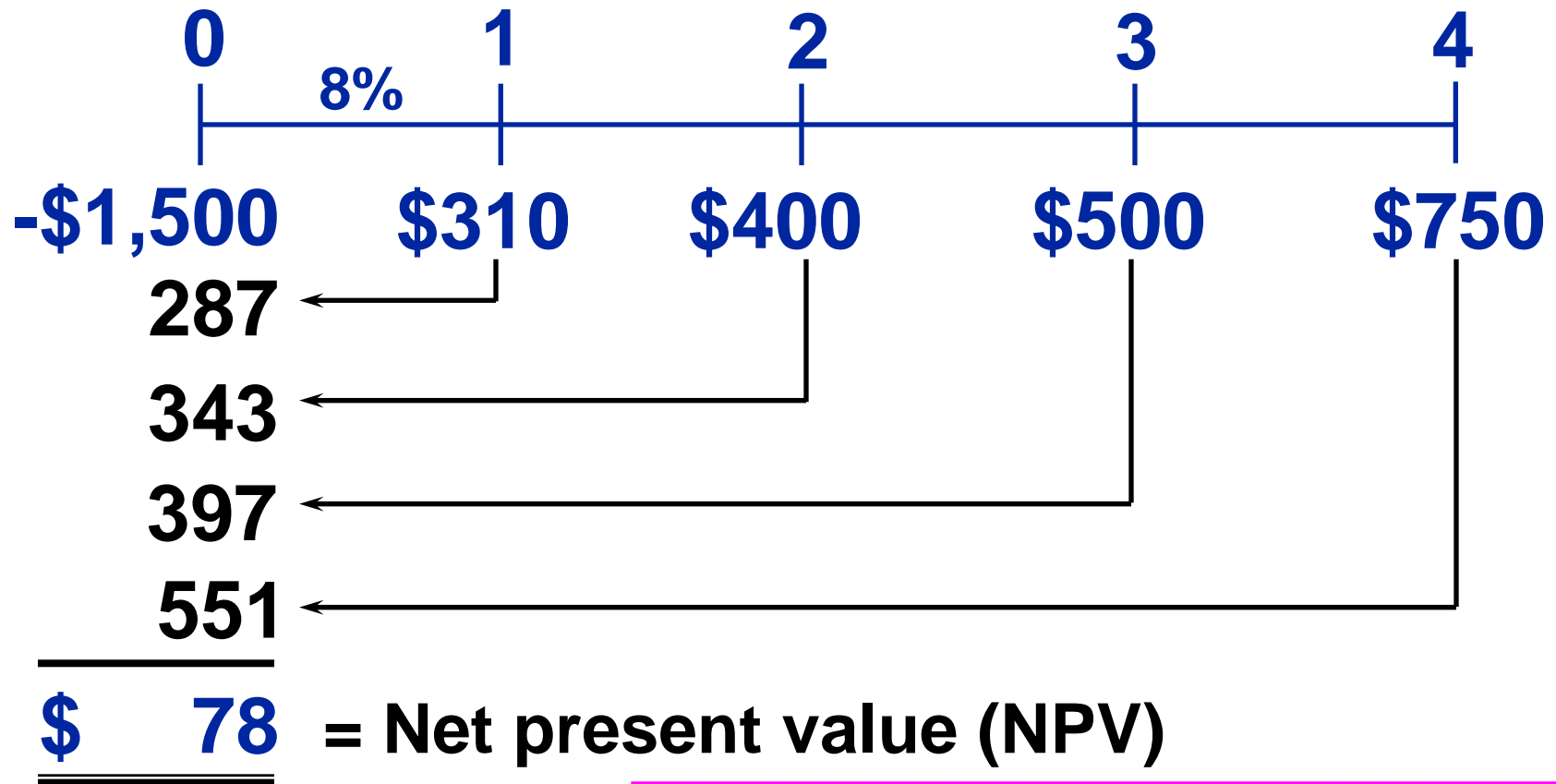
? Where do these numbers come from?

Simple Dollar Return



? Is this a good measure?

Discounted Cash Flow (DCF) Dollar Return



? Where did the 8% come from?

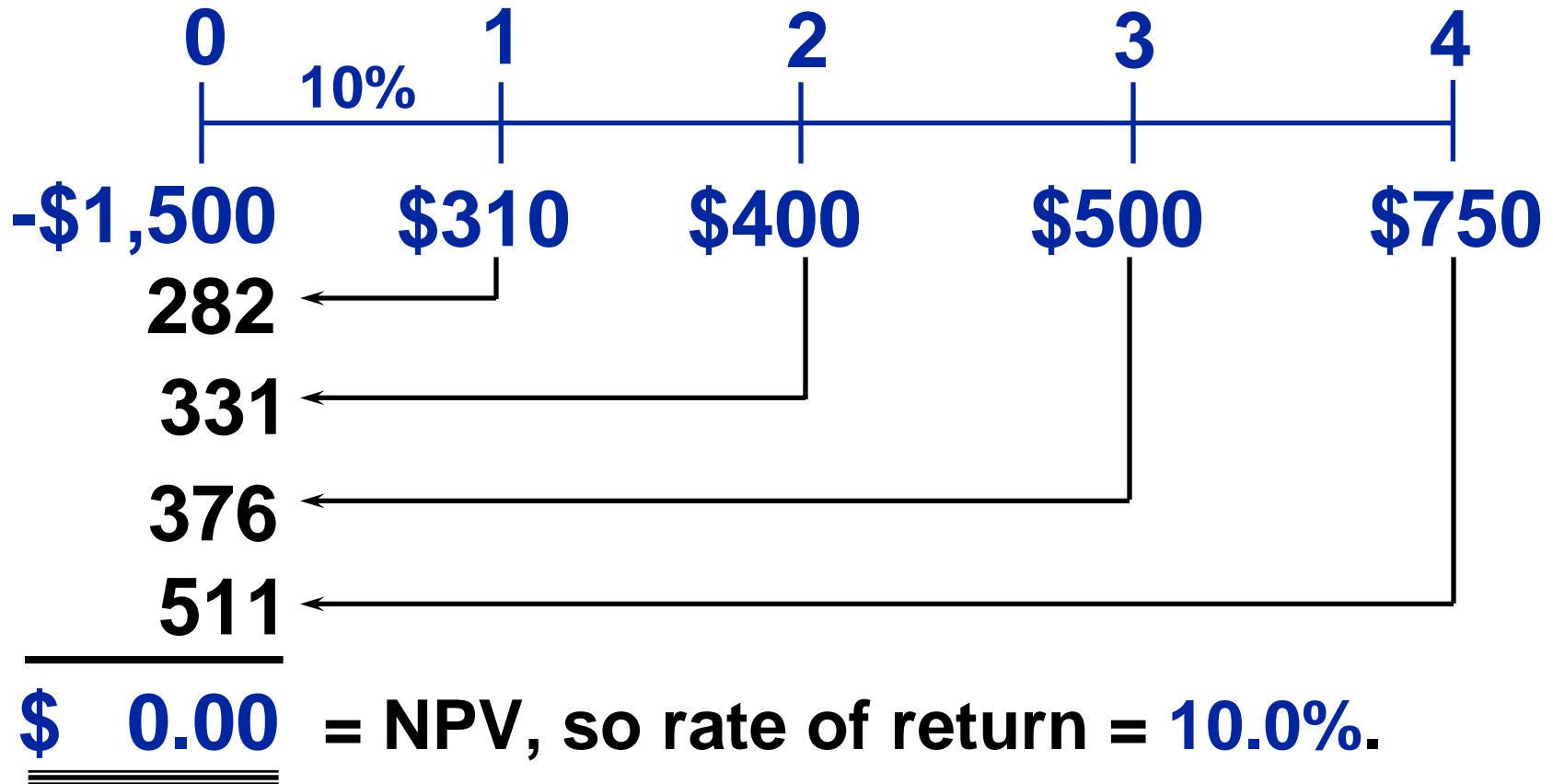
Spreadsheet Solution

	A	B	C	D
1				
2	8.0%	Rate	Interest rate	
3	\$ (1,500)		Year 0 CF	
4	310	Value 1	Year 1 CF	
5	400	↑	Year 2 CF	
6	500	↓	Year 3 CF	
7	750	Value 1	Year 4 CF	
8				
9				
10	\$ 78	=NPV(A2,A4:A7)+A3 (entered into Cell A10)		

DCF Dollar Return (Cont.)

- The key to the effectiveness of this measure is that the discounting process automatically recognizes the *opportunity cost of capital*.
- An NPV of zero means the project just earns its opportunity cost rate.
- A positive NPV indicates that the project has positive financial value after opportunity costs are considered.

Rate of (Percentage) Return



Spreadsheet Solution

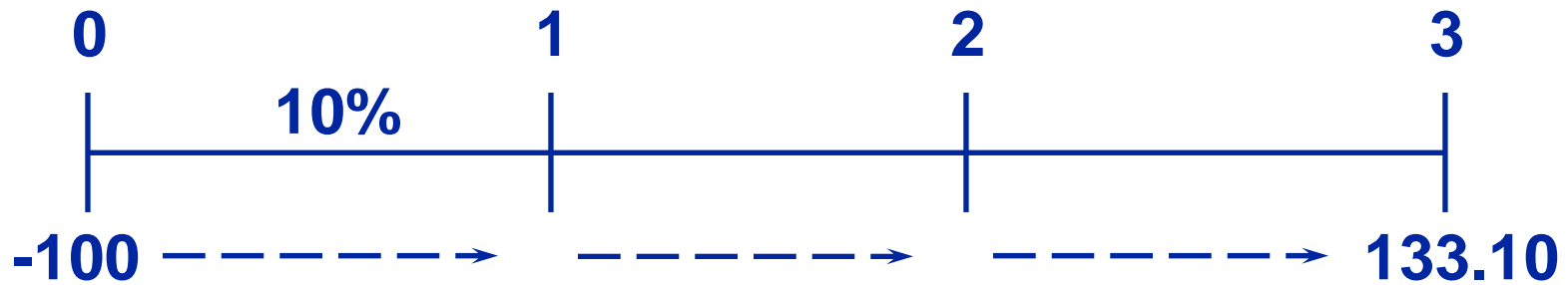
	A	B	C	D
1				
2	8.0%	Rate	Interest rate	
3	\$ (1,500)	Values	Year 0 CF	
4	310	↑	Year 1 CF	
5	400	↓	Year 2 CF	
6	500	↓	Year 3 CF	
7	750	Values	Year 4 CF	
8				
9				
10	10.0%	=IRR(A3:A7,A2) (entered into Cell A10)		

Rate of Return (Cont.)

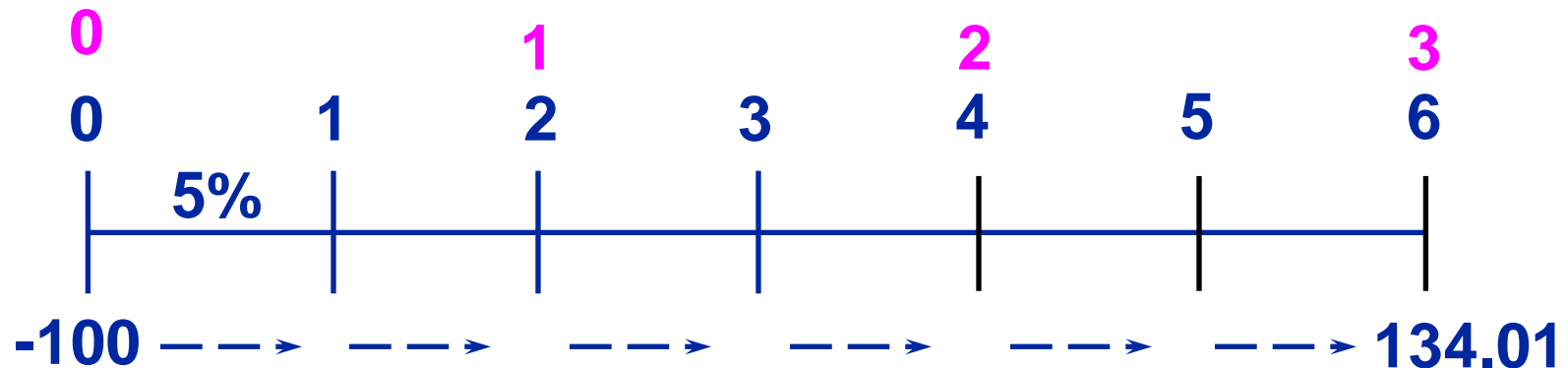
- In capital investment analyses, the rate of return often is called **internal rate of return (IRR)**.
- In essence, it is the percentage return expected on the investment.
- To interpret the rate of return, it must be ***compared*** with the opportunity cost of capital, in this case **10%** versus **8%**.

Intrayear Compounding

- Thus far, all examples have assumed annual compounding.
- When compounding occurs intrayear, the following occurs:
 - Interest is earned on interest during the year (more frequently).
 - The *future value* of an investment is *larger* than under annual compounding.
 - The *present value* of an investment is *smaller* than under annual compounding.



Annual: $FV_3 = 100 \times (1.10)^3 = 133.10$.



Semiannual: $FV_6 = 100 \times (1.05)^6 = 134.01$.

Effective Annual Rate (EAR)

- EAR is the *annual rate* that causes the PV to grow to the same FV as under intrayear compounding.
- What is the EAR for **10%**, semiannual compounding?
 - Consider the FV of **\$1** invested for one year. $FV = \$1 \times (1.05)^2 = \mathbf{\$1.1025}$.
 - EAR = **10.25%**, because this rate would produce the same ending amount (**\$1.1025**) under annual compounding.

The EAR Formula

$$\text{EAR} = \left(1 + \frac{I_{\text{Stated}}}{M} \right)^M - 1.0$$

$$= \left(1 + \frac{0.10}{2} \right)^2 - 1.0$$

$$= (1.05)^2 - 1.0 = 0.1025 = \mathbf{10.25\%}.$$

Or, use the *EFF%* key on a financial calculator.

EAR of 10% at Various Compounding

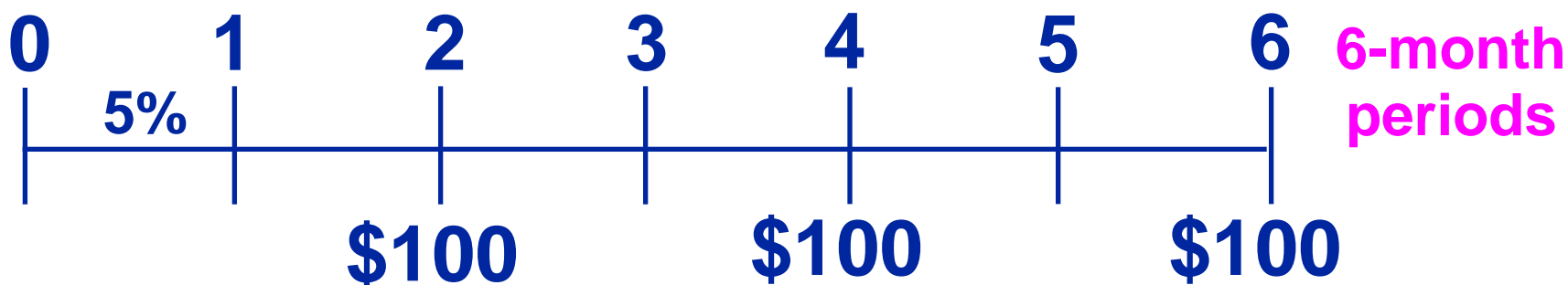
$$EAR_{\text{Annual}} = 10\%$$

$$EAR_Q = (1 + 0.10/4)^4 - 1.0 = 10.38\%$$

$$EAR_M = (1 + 0.10/12)^{12} - 1.0 = 10.47\%$$

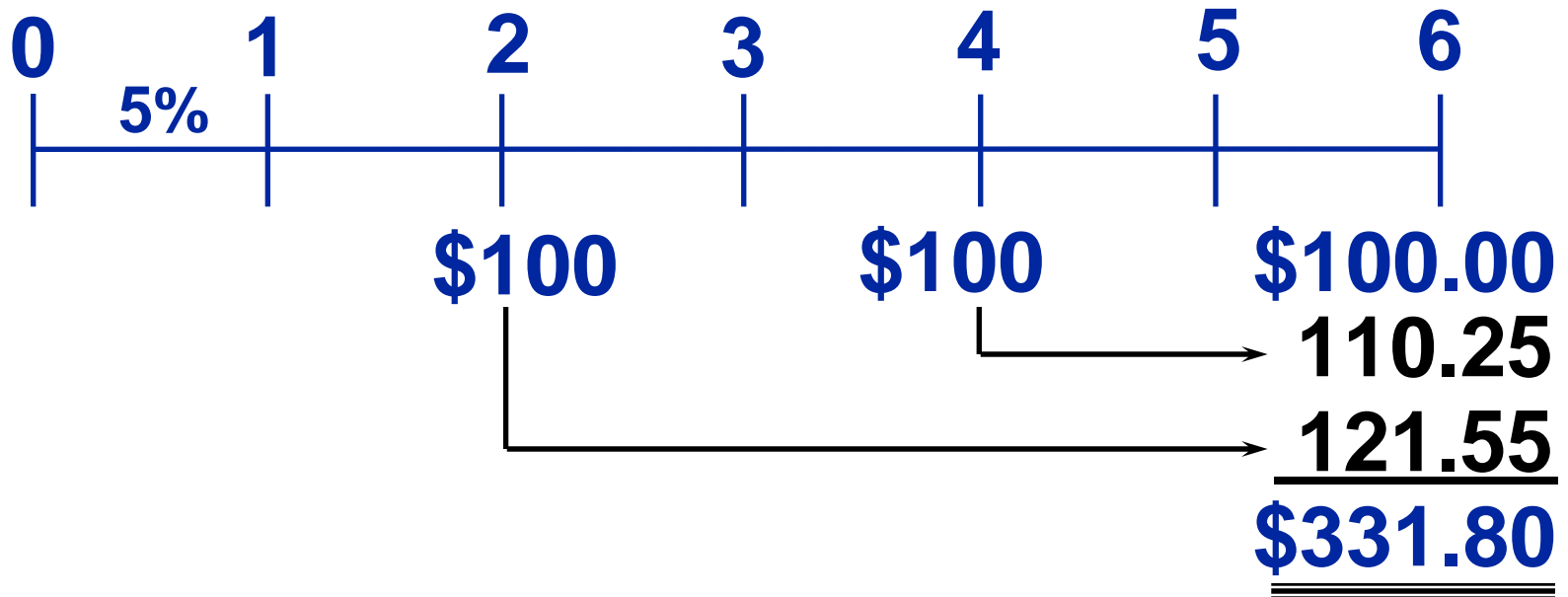
$$EAR_{D(360)} = (1 + 0.10/360)^{360} - 1.0 = 10.52\%$$

Using the EAR



Here, payments occur *annually*, but compounding occurs *semiannually*, so we can not use normal annuity valuation techniques.

First Method: Compound Each CF



Second Method: Treat as an Annuity

- Find the EAR for the stated rate:

$$\text{EAR} = \left(1 + \frac{0.10}{2} \right)^2 - 1 = \mathbf{10.25\%}.$$

- Then use standard annuity techniques:

INPUTS

3

10.25

0

-100

N

I/YR

PV

PMT

FV

OUTPUT

331.80

Amortization

Construct an **amortization schedule** for a **\$1,000**, **10%** annual rate loan with **three** equal payments.

Step 1: Find the required payments.



INPUTS

3	10	-1000		0
N	I/YR	PV	PMT	FV

OUTPUT

402.11

Step 2: Find interest charge for Year 1.

$$\text{INT}_t = \text{Beginning balance} \times i.$$

$$\text{INT}_1 = \$1,000 \times 0.10 = \mathbf{\$100}.$$

Step 3: Find repayment of principal in Year 1.

$$\begin{aligned} \text{Repmt} &= \text{PMT} - \text{INT} \\ &= \$402.11 - \$100 \\ &= \mathbf{\$302.11}. \end{aligned}$$

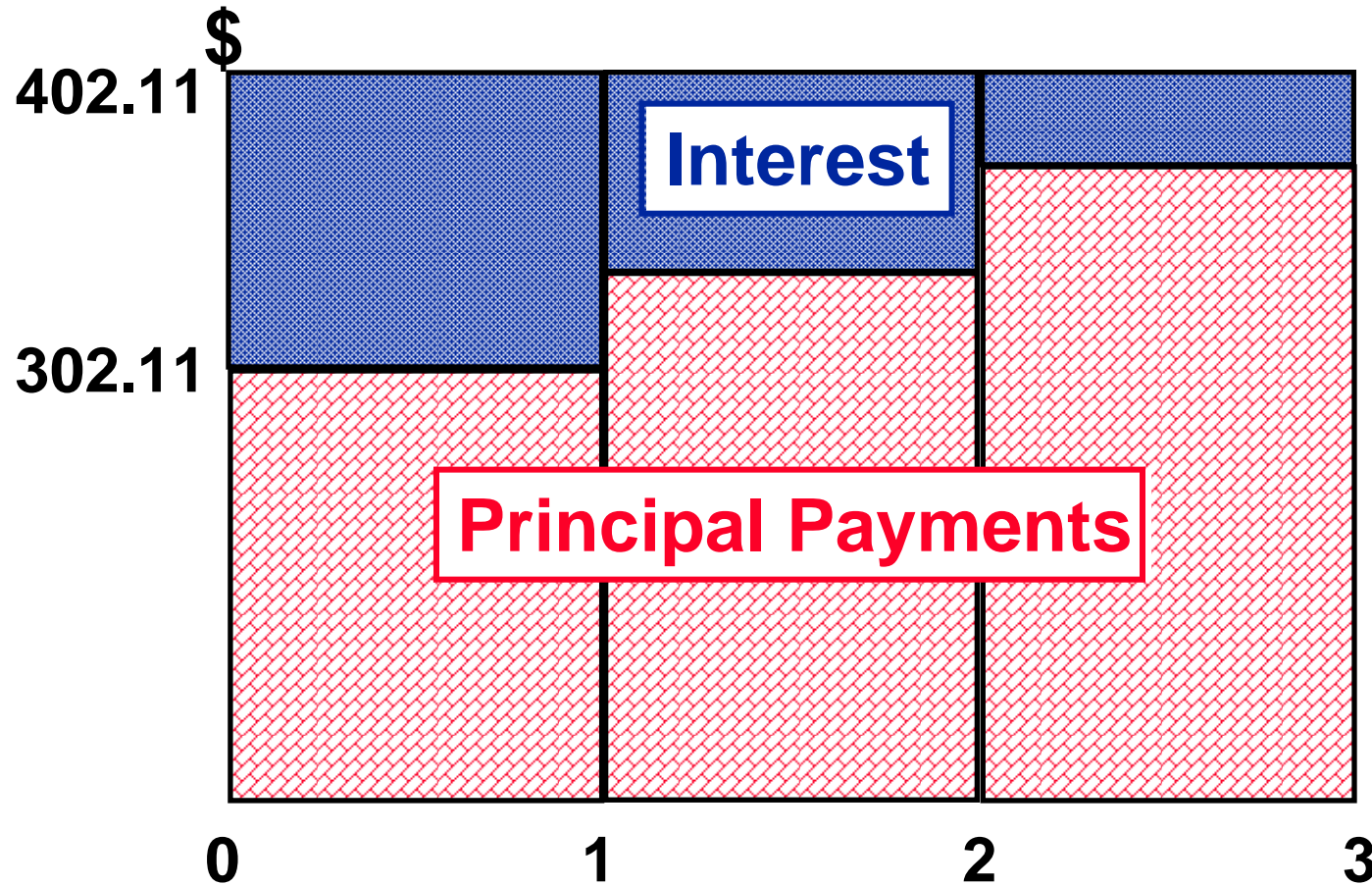
Step 4: Find ending balance at end of Year 1.

$$\begin{aligned}\text{End bal} &= \text{Beg balance} - \text{Repayment} \\ &= \$1,000 - \$302.11 = \mathbf{\$697.89}.\end{aligned}$$

Repeat these steps for Years 2 and 3 to complete the amortization table.

YR	BEG BAL	PMT	INT	PRIN PMT	END BAL
1	\$1,000	\$402	\$100	\$302	\$698
2	698	402	70	332	366
3	366	402	37	366	0
TOTAL		\$1,206.34	\$206.34	\$1,000	

Note that **annual interest declines** over time while the **principal payment increases**.



Level payments. Interest declines because outstanding balance declines. Lender earns 10% on loan outstanding, which is falling.

Conclusion

- **This concludes our discussion of *Chapter 9* (Time Value Analysis).**
- **Although not all concepts were discussed in class, you are responsible for all of the material in the text.**
- ? **Do you have any questions?**